

RACES AND GAMES

Races and games are also related to time, speed and distance. Races and games problems are asked in various competitive exams.

To solve the problems of races and games we should understand the terms given below.

Race: Race is a competition in which participants try to cover a fixed distance in the least possible time. Race may involve running, riding, driving, rowing etc.

Race course: A path on which a race takes place is called a race course *e.g.* ground, road, swimming pool etc.

Starting point: A point from where the race starts is called the starting point.

Finish point or Winning point or Goal: A point where race finishes is called the finish point or winning point or goal.

Winner: A person who first reaches the finish point is called winner.

Dead heat: If all the participants reach the finish point at the same instant of time, then this situation is called dead heat.

Suppose A and B are two participants in a race.

"A gives B a start of x metres" means A starts the race from the starting point whereas B starts the race x metres ahead of A *i.e.* to cover a distance of 100 m, A will cover 100 m while B will cover $(100 - x)$ m.

"A beats B by x metres" means when A reached the finish point, B is behind A by x metres *i.e.* in a 100 m race, A has covered 100 m while B has covered only $(100 - x)$ m.

"A gives B a start of t minutes" means A will start t minutes after B starts from the starting point but both A and B will reach the finishing point at the same time.

"A beats B by x metres and t minutes" means A and B start from the starting point at the same instant, but while A reaches the finishing point, B is x metres behind the finishing point and B takes t minutes more as compared to A to complete the race *i.e.* B covers the remaining x metres in extra t minutes. So, speed of B is $\frac{x}{t}$ m/min.

Games: A game is a structured form of play. Games are sometimes played only for enjoyment, sometimes for achievement or reward. Games can be played alone or in teams.

Winning or losing a game is mainly based on points, goals or runs scored by a player or a team.

A game of 100 means that the person among the participants who scores 100 points first is the winner.

Suppose A and B are two participants in a game. A scores 100 points while B scores only 70 points, then we say that **"A beat B by 30 points"** or **"A can give B 30 points"**.

ILLUSTRATIVE EXAMPLES

Example 1. A can run 22.5 m while B runs 25 m in the same time. In a 1000 m race, find by how much distance B beats A?

Solution. When B runs 25 m, A runs = 22.5 m

$$\text{When B runs 1 m, A runs} = \frac{22.5}{25} \text{ m}$$

and when B runs 1000 m, A runs = $\frac{22.5}{25} \times 1000 \text{ m} = 900 \text{ m}$.

Hence, B beats A by 100 m.

Example 2. In a 600 m race, A beats B by 37.5 m or by 10 seconds, find the time taken by B to finish the race.

Solution. A beats B by 37.5 m or by 10 seconds means B covers a distance of 37.5 m in 10 seconds.

$$\therefore \text{Speed of B} = \frac{37.5}{10} = 3.75 \text{ m/sec.}$$

So, B covers 600 m in $\frac{600}{3.75}$ seconds = 160 seconds.

Hence, the time taken by B to finish the race is 160 seconds.

Example 3. In a 500 m race, A reaches the finish point in 20 seconds and B reaches in 25 seconds. By how much distance A beats B?

Solution. Speed of B = $\frac{500}{25} = 20 \text{ m/sec}$

We will find the distance covered by B after A has finished the race.

Distance covered by B in (25 – 20) seconds i.e. in 5 seconds = $20 \times 5 = 100 \text{ m}$

Hence, A beats B by 100 m.

Example 4. In a 500 m race, the ratio of speeds of two participants A and B is 4 : 5 respectively. If A has a start of 180 m, then find the distance by which A wins.

Solution. Since A has a start of 180 m.

It means A will have to cover a distance of $500 - 180$ i.e. 320 m.

Given ratio of speeds of A and B = 4 : 5

$\Rightarrow$ when A covers 4 m, B covers = 5 m

$\Rightarrow$ when A covers 1 m, B covers = $\frac{5}{4} \text{ m}$

$\Rightarrow$ when A covers 320 m, B covers = $\frac{5}{4} \times 320 \text{ m} = 400 \text{ m}$

Hence, A wins by 100 m.

 **Example 5.** A runs 3 times as fast as B. If A gives B a start of 50 m. Find how far must be the finish point on the race course so that A and B reach the goal at the same time?

Solution. Given A runs 3 times as fast as B.

It means ratio of speeds of A and B = 3 : 1

$\Rightarrow$ when A covers 3 m, B covers 1 m

or we can say that A gains (3 – 1) m i.e. 2 m by B in a race of 3 m.

Given, A gives B a start of 50 m means A gains 50 m by B in the race.

Thus, when A gains 2 m by B, the race is of 3 m

when A gains 1 m by B, the race is of $\frac{3}{2} \text{ m}$

when A gains 50 m by B, the race is of $\frac{3}{2} \times 50 \text{ m} = 75 \text{ m}$

Hence, length of race = 75 m.

Example 6. In a 100 m race, A can give a start of 10 m to B and a start of 28 m to C. In the same race, how much start can B give to C?

Solution. In the same time, A covers 100 m, B covers (100 – 10) m i.e. 90 m and C covers (100 – 28) m i.e. 72 m.

If the race is between B and C only,
when B covers 90 m, C covers = 72 m.

When B covers 1 m, C covers = $\frac{72}{90}$ m

When B covers 100 m, C covers = $\frac{72}{90} \times 100$ m = 80 m.

Hence, B can give a start of 20 m to C.

Example 7. In a 10 km race, A, B and C each running at uniform speed get the gold, silver and bronze medals respectively. If A beats B by 1 km and B beats C by 1 km, then find by how many metres does A beat C?

Solution. A beats B by 1 km means,
in the same time, A travels 10 km and B travels 9 km

i.e. Speed of A : Speed of B = 10 : 9 = 100 : 90

Similarly, B beats C by 1 km means,

speed of B : speed of C = 10 : 9 = 90 : 81

$\therefore$ Speed of A : Speed of B : Speed of C = 100 : 90 : 81

It means, in the same time

A travels 100 m and C travels 81 m

$\Rightarrow$ A travels 10000 m and C travels 8100 m

$\Rightarrow$ A beats C by 1900 m.

Example 8. In a 900 metres race, A gives B a start of 150 metres and defeats him by 50 seconds. If the speed of A is 4.5 m/sec, then find the speed of B.

Solution. A gives B a start of 150 metres means A covers 900 m and B covers (900 – 150) m i.e. 750 m.

Given speed of A = 4.5 m/sec.

$\therefore$ Time taken by A to cover 900 m = $\frac{900}{4.5} = 200$ sec.

Since A defeats B by 50 seconds, it means B covers a distance of 750 m in 200 + 50 i.e. 250 seconds.

So, speed of B = $\frac{750}{250} = 3$ m/sec.

Example 9. In a game of 100 points, A can give 20 points to B and 28 points to C. Find how many points can B give C?

Solution. According to given,

when A scored 100 points, B scored (100 – 20) i.e. 80 points and C scored (100 – 28) i.e. 72 points.

When B scored 80 points, C scored = 72 points

When B scored 100 points, C scored = $\frac{72}{80} \times 100 = 90$ points

Hence, B can give 10 points to C.

Example 10. In a game, A can give 20 points to B, A can give 32 points to C and B can give 15 points to C. How many points make the game?

Solution. Let x points make the game, then

A scores x points, B scores $(x - 20)$ points and C scores $(x - 32)$ points

B scores x points, C scores $(x - 15)$ points

$$\therefore \frac{x-20}{x-32} = \frac{x}{x-15} \Rightarrow x^2 - 35x + 300 = x^2 - 32x$$

$$\Rightarrow 3x = 300 \Rightarrow x = 100$$

Hence, 100 points make the game.

EXERCISE 1.5

1. A can run 40 metres while B run 50 metres in the same time. In a 1000 m race, find by how much distance B beats A?
2. In a 100 m race, A covers a distance in 36 seconds and B in 45 seconds. Find by how much distance A beats B?
3. In a 200 m race, A beats B by 5 m or 3 seconds. Find A's time to cover the race course.
4. A runs 4 times as fast as B. If A gives B a start of 60 m, how far must the goal on the race course be so that A and B reach it at the same time?
5. A and B participate in 200 m race. A runs at the speed of 6 km/h. A gives B a start of 18 m and still beats him by 6 seconds. Find the speed of B in km/h.
6. In a 500 m race, the ratio of the speeds of two participants A and B is 3 : 4 respectively. If A has a start of 140 m, then find the distance by which A wins.
7. In a 100 m race, A beats B by 20 m and C by 28 m. Find by how much distance B will beat C in a race of 500 m?
8. In a 100 m race, A can beat B by 20 m and B can beat C by 10 m. In the same race, find by how much distance does A beat C?
9. In a 200 m race, A can give a start of 18 m to B and a start of 31 m to C. In a race of 350 m, how much start can B give to C?
10. In a flat race, A beats B by 15 m and C by 29 m. When B and C run over the course together B wins by 15 m. Find the length of the race course.
11. A can run 1 km in 4 minutes 54 seconds and B in 5 minutes. How many metres start can A give B in 1 km race so that the race may end in dead heat?
12. In a race of 600 m, A can beat B by 60 m and in a race of 500 m B can beat C by 50 m. By how much metres will A beat C in a race of 400 m?
13. In a game of 90 points, A can give 15 points to B and 30 points to C. How many points can B give C in a game of 100 points?
14. In a game, A can give 20 points to B in 60 and 18 points to C in 90. How many points can C give B in a game of 120?
15. In a game, A can give 15 points to B, A can give 30 points to C and B can give 20 points to C. How many points make the game?

Answers

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| 1. 200 m | 2. 20 m | 3. 117 seconds | 4. 80 m |
| 5. 5.2 km/h | 6. 20 m | 7. 50 m | 8. 28 m |
| 9. 25 m | 10. 225 m | 11. 20 m | 12. 76 m |
| 13. 20 points | 14. 20 points | 15. 60 points | |